# Abstracts

## Saba ALIYARI

### Difference Tropical Geometry

Generally speaking by a difference field we mean a field with an automorphism. A valued difference field is a valued field $(K, v)$ together with a field automorphism $\sigma$. This automorphism induces a group automorphism $\hat{\sigma}$ on the value group $\Gamma$ such that $\forall v(x) \in \Gamma$, $\hat{\sigma}(v(x)) = v(\sigma(x))$. Depending on the interaction of $v$ with $\sigma$ we may have different cases for valued difference fields. As in our work the value group is supposed to be $\mathbb{R}$, the case we are interested in here is called a multiplicative valued difference field in which we have

$$\forall x \in K, \quad v(\sigma(x)) = \rho.v(x)$$

where $\rho \in \mathbb{R}_{>0}$.

A difference polynomial $f$ is an element of $K[x, \sigma(x), \sigma^2(x), \ldots]$. In order to have a better understanding of these polynomials and their roots we decided to apply tropical geometry. One of the main theorems which leads to a better understanding of the roots of a polynomial is Kapranov's theorem. We proved a difference version of this theorem.

## Nicholas ANDERSON

### Tropically Realizable Matroids

A tropical variety $L$ is said to be tropically realizable if there is a tropical ideal $I$ for which $V(I) = L$. In this talk, we will examine the case where $L$ is a tropical linear space with weights equal to 1, i.e. when $L$ is the variety of linear tropical polynomials. Tropical linear spaces correspond to what are called valuated matroids, and thus when a tropical linear space is tropically realizable, we will say its associated matroid is tropically realizable. We will see that the class of tropically realizable matroids exactly corresponds to the class of valuated matroids that have symmetric products of arbitrarily large size.

Dorian BERGER

## Étale morphisms between Berkovich spaces over $\mathbb{Z}$

The Berkovich geometry allows the construction of analytic spaces over any Banach ring. In particular, one can consider analytic spaces over $\mathbb{Z}$ equipped with the usual absolute value. In this case, one obtains spaces naturally fibered with complex and $p$-adic analytic spaces. In this talk, we will discuss étale morphisms between such spaces, which induce local isomorphisms between complex fibers and usual étale morphisms between $p$-adic fibers. We will show how to prove the fiber criterion for those morphisms. The proof works for any valued field, rings of integers of a number field and discrete valuation rings.

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Loujean COBIGO

## Tropical Spin Hurwitz numbers

Classical Hurwitz numbers count the number of branched covers of a fixed target curve that exhibit a certain ramification behaviour. It is an enumerative problem deeply rooted in mathematical history. A modern twist: Spin Hurwitz numbers were introduced by Eskin-Okounkov-Pandharipande in 2008 for certain computations in the moduli space of differentials on a Riemann surface. Similarly to Hurwitz numbers they are defined as a weighted count of branched coverings of a smooth algebraic curve with fixed degree and branching profile. In addition, they include information about the lift of a theta characteristic of fixed parity on the base curve. In this talk we investigate them from a tropical point of view. We start by defining tropical spin Hurwitz numbers as result of an algebraic degeneration procedure, but soon notice that these have a natural place in the tropical world as tropical covers with tropical theta characteristics on source and target curve. Our main results are two correspondence theorems stating the equality of the tropical spin Hurwitz number with the classical one. We obtain concrete computational results in special cases that carry over to the classical world via our correspondence.

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Yanbo FANG

## Non-Archimedean critical Fubini–Study metric

Over a non-Archimedean field, the (local) projective height of a projective variety X embedded in $\mathbb{P}^n$ by ample line bundle $L$ equipped with a Fubini–Study metric is given by the induced Gauss norm of its Chow form. Under the natural action by $SL(n+1)$ on $H^0(X, L)$, a F-S metric $h$ on $L$ is called Chow critical if it minimises this height among other F-S metrics in the same orbit.

We calculate the corresponding Euler-Lagrange condition using either analytic geometry (with intersection line bundle) or tropical geometry (with incidence correspondence), and express the result in terms of the Monge-Ampère polytope (a polymatroid) of $(X, L, h)$.

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Andreas GROSS

## The tropical cocycle class map and a tropical Koszul complex

The tropical cycle class map sends tropical cycles to their associated homology classes. In case the underlying space is smooth, both tropical cycles and homology classes can be intersected and it has been expected that the cycle class map respects these intersection operations. To prove this in general, we work on the level of cocycles and define a cocycle class map that sends tropical cocycles to their associated tropical cohomology class. The key ingredient for this is a modification of a Koszul complex that links the combinatorial Chow groups appearing in the study of combinatorial Hodge theory to the Orlik-Solomon-algebras appearing in the definition of tropical (co)homology.

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Trevor GUNN

## Multiplicities over hyperfields in one and multiple variables

We will introduce the notion of a hyperfield and explain how factoring univariate polynomials over the tropical hyperfield and hyperfield of signs recovers, respectively, the Newton polygon rule (relating the valuations of the roots to those of the coefficients) and Descartes' rule of signs (relating signs of roots to signs of coefficients). After, we will turn our attention to zero-dimensional systems of polynomials in multiple variables where using sparse resultants we explain on the tropical side, how this is connected to the Bernstein–Khovanskii–Kushnirenko (BKK) bound (which says that the number of roots in $(\mathbb{C}^*)^n$ is bounded by the mixed volume of the Newton polytopes) and on the real algebraic side, we will explore some questions and bounds on the number of roots in a given orthant in terms of the signs of the original polynomials and/or resultant polynomial. This is joint work with Andreas Gross.

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Paul Alexander HELMINCK

## Generic root counts and flatness in tropical geometry

A famous theorem by Bernstein, Kushnirenko and Khovanskii (the BKK theorem) says that the generic root count of a generic square polynomial system is the mixed volume of the corresponding Newton polygons. In this talk we give a generalization of this theorem using methods from tropical and non-archimedean geometry. We show that this generalization can be used to give root counts that improve the BKK root count. An important role in this generalization is played by the new notion of tropical flatness. We prove an analogue of Grothendieck's generic flatness theorem, showing that morphisms are tropically flat over an open dense subset of the base space.

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Matthias HIPPOLD

## Hurwitz Space in the Case of Wild Ramification

Hurwitz Spaces are moduli spaces parametrising covers of curves with fixed ramification profile. While they are very well understood in characteristic zero, this is not true in positive characteristic, especially in the case of wild ramification when the characteristic of the ground field divides the ramification indices. We will introduce such a Hurwitz space in the case of characteristic 2, parametrizing 2-to-1-covers of the projective line by elliptic curves fully ramified over 4 points. For this description, we will introduce the concept of Berkovich skeleta of a curve and explain how they constitute a tropicalization of this Hurwitz space. Finally, we will give a modular interpretation of the constructed space using a log-functor and explain how we expect this data to constitute a nice moduli functor in the general setting.

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Hernan IRIARTE

## Higher rank tropical geometry

We study the tropicalization of algebraic varieties with respect to valuations with values in $\mathbb{R}^k$. In this context, an algebraic variety is given by a fibration in which the base and each fiber are tropical varieties in the usual way. The fibration admits the structure of a polyhedral complex with coefficients in the ordered ring $\mathbb{R}[x]/(x^k)$. In the case of hypersurfaces, we can understand completely the combinatorics of this fibration from a layered regular subdivision of the Newton polytope.

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Andrés JARAMILLO PUENTES

## Enriched tropical intersection

Tropical geometry has been proven to be a powerful computational tool in enumerative geometry over the complex and real numbers. In this talk we present an example of a quadratic refinement of this tool, namely a proof of the quadratically refined Bézout's theorem for tropical curves. We recall the necessary notions of enumerative geometry over arbitrary fields valued in the Grothendieck-Witt ring. We will mention the Viro's patchworking method that served as inspiration to our construction based on the duality of the tropical curves and the refined Newton polytope associated to its defining polynomial. We will prove that the quadratically refined multiplicity of an intersection point of two tropical curves can be computed combinatorially. We will use this new approach to prove an enriched version of the Bézout theorem and of the Bernstein–Kushnirenko theorem, both for enriched tropical curves. Based on a joint work with S. Pauli.

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Soham KARWA

## Non-Archimedean SYZ and Periods

The SYZ conjecture of Mirror Symmetry roughly states that Calabi-Yau varieties come in pairs and they admit dual special Lagrangian torus fibrations over the same base with a discriminant locus of codimension $\geq 2$. In this talk, we will see how one can reformulate this conjecture in the world of Berkovich spaces via non-archimedean torus fibrations. Using this framework, we will then compute the periods of degenerations of Calabi-Yau varieties and discuss interesting applications of this idea.

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Patrick KENNEDY-HUNT

## The logarithmic Hilbert scheme and its tropicalisation

Let $X$ be a toric variety with toric boundary $D$. We study subschemes $Z$ of $X$ whose intersection with $D$ satisfies a transversality condition. The logarithmic Hilbert scheme is a proper moduli space of subschemes $Z$ in expansions of $X$ which satisfy our transversality condition. The tropicalisation of the logarithmic Hilbert scheme is a moduli problem on the category of cones. The key insight in our construction is a tropical criterion for when $Z$ satisfies this transversality condition and how this behaves in families. A particularly beautiful example is the logarithmic linear system: a toric modification of projective space. The associated fan is closely related to the geometry of tropical curves and secondary polytopes. Joint work with Dhruv Ranganathan.

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Tudor MICU

## The structure of special fibres through valuations

A normal model $\mathcal{X}$ of a projective curve $X$ over a discrete valued field $K$ can be identified with a finite set $V(X)$ of valuations on the function field $K(X)$ of the curve. Each of these valuations corresponds to an irreducible component of the special fibre of $\mathcal{X}$. When the curve $X$ is the projective line, these valuations can be described explicitly as finite sequences $(\phi_i, \lambda_i)_{i=1,\ldots,n}$, where $\phi_i \in K[t]$ and $\lambda_i \in \mathbb{Q}$. This allows for a compact description of the special fibre, without the use of affine charts. In fact, $V(X)$ can be identified with a subset of the Berkovich projective line $\mathbb{P}^1_{Berk}(K)$, whose structure enables us to answer the following questions:

1. Which components of the special fiber of $\mathcal{X}$ intersect?
2. Given a closed point $x \in X$, which irreducible components of the special fiber of $\mathcal{X}$ does $x$ reduce to?

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Simon MOE

## Stable rationality of lattice polytopes

To a lattice polytope one associates a toric variety and an ample line bundle. One may call a polytope (stably) rational if these hypersurfaces are (stably) rational. Nicaise–Ottem used degeneration techniques to study stable rationality of such hypersurfaces in terms of degenerations arising from polyhedral subdivisions of the polytope.

This talk will introduce the concept of stably rational polytopes and combinatorial degeneration techniques. We focus on the question of whether stable irrationality is preserved under inclusion of polytopes, and give some low dimensional answers. If time we'll also discuss some problems relating to the geometry of empty simplices.

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Felix ROEHRLE

## A tropical Version of Donagi's $n$-gonal construction

Given a degree 2 cover of smooth algebraic curves $\tilde{C} \to C$ with $C$ being $n$-gonal, Donagi's construction associates a new double cover $\tilde{C}' \to C'$ with $C'$ being $2^n$-gonal. For $n = 2$, 3, or 4 there are close relations between Prym varieties and/or Jacobian varieties of the input and the output of the construction. In this project we describe an analog of the $n$-gonal construction for tropical curves instead of algebraic curves. Our goal is to obtain theorems in parallel to the algebraic situation. This is joint work in progress with Dmitry Zakharov.

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Uriel SINICHKIN

## Refined tropical invariants in positive genus

Gottsche-Schroeter refined invariants are tropical invariants which depend on a formal parameter $y$, and specialize to rational descendant Gromov-Witten invariants for $y = 1$ and to broccoli invariants for $y = -1$. Schroeter and Shustin introduced a version of those invariants in genus 1, but the significance of their values at $y = 1$ and $y = -1$ have remained unknown. In this talk we will discuss a more local description of Schroeter-Shustin invariants and present a complex algebro-geometric meaning for their values at $y = 1$. If time permits, we will discuss possible generalizations to higher genus. This talk is based on joint work with Eugenii Shustin.

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Yuki TSUTSUI

## On graded modules associated with line bundles on tropical curves and integral affine manifolds

In the talk, we will discuss graded modules associated with line bundles on tropical curves and integral affine manifolds. These graded modules are defined using ideas from mirror symmetry and the Strominger–Yau–Zaslow conjecture. For tropical curves and integral affine manifolds with a Hessian metric, we give a formula for the Euler numbers of these graded modules, which is (closely related but) different from Riemann–Roch theorems for graphs and tropical curves by Baker–Norine and Gathmann–Kerber.

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Alejandro VARGAS

## A moduli space of tropical covers that witness the gonality bound of metric graphs

A tropical cover $\phi\colon \Gamma \to \Delta$ is a morphism of metric graphs such that the count of points in every fibre, using a natural multiplicity, is a constant $d_\phi \in \mathbb{Z}_{\geq 1}$, called the degree of $\phi$, and the ramification divisor $R_\phi = K_\Gamma - \phi^* K_\Delta$ is effective. Tropical covers play a role in tropical Brill-Noether theory, tropical Hurwitz numbers, divisor theory of graphs, and so on.

The tree-gonality $gon(\Theta)$ of a metric graph $\Theta$ is the minimum degree of a tropical cover from a tropical modification of $\Theta$ to a metric tree $\Delta$. This definition mirrors the classical gonality of a Riemann surface X, i.e. min degree of a map from $X$ to $\mathbb{P}^1$, which roughly measures the complexity of $X$.

In both instances, the gonality is upper bounded by $\lceil g/2 \rceil + 1$, where $g$ is the genus of $X$, or respectively of $\Gamma$. On the tropical side, this bound was first proved in work by Baker and later Caporaso, who developed machinery to transport the classical result to the tropical world. A purely combinatorial proof was given later by Draisma and Vargas, which relies on giving some explicit constructions and a deformation procedure to obtain the maps that witness the gonality bound. Building on the latter, this talk describes how those gonality-witnessing maps can be put into a moduli space, and how when $g$ is even this moduli space itself is a higher dimensional tropical cover of $\mathscr{M}_g^{trop}$ whose degree is the $g/2$-th Catalan number, again mirroring a classical result in Brill-Noether theory.

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Art WAETERSCHOOT

## Pullback of weight functions and logarithmic differents

Given a morphism of smooth varieties over a non-archimedean discretely valued field, the change of the weight functions on the Berkovich analytifications is related to a logarithmic different of the residue fields. This can be applied to study Berkovich skeleta of curves under wildly ramified base change.

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Yuto YAMAMOTO

## Tropical varieties and integral affine manifolds with singularities

There are two types of spaces which we study in tropical geometry. One is tropical varieties which appear as the tropicalizations of algebraic varieties over a valuation field. The other one is integral affine manifolds with singularities which arise as the dual intersection complexes of toric degenerations in the Gross–Siebert program. In the talk, we discuss relations between these two different types of tropical spaces. We construct contraction maps from tropical Calabi–Yau varieties to corresponding integral affine manifolds with singularities, and show that they preserve tropical (co)homology groups and the invariants of tropical structures called eigenwaves/radiance obstructions.