Valuated Matroids, Tropicalized Linear Spaces and the Affine Building of $PGL_{r+1}(K)$

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slides available at https://vargas.page/
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joint with: Battistella, Kühn, Kuhrs, and Ulirsch

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Thus, a central question is what information survives this degeneration, and how to reconcile all choices done through the process

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Crucial: Description Ⓓ suggests Trop is a **projection** of a space of semivaluations, a gigantic object.

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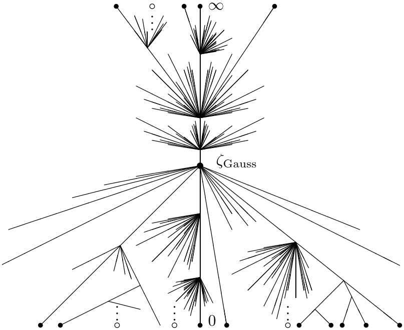

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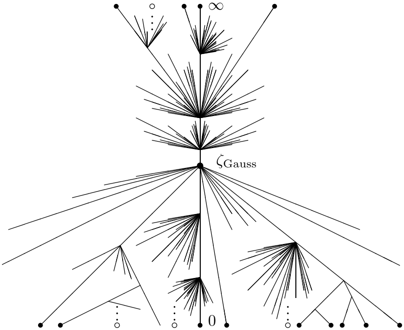

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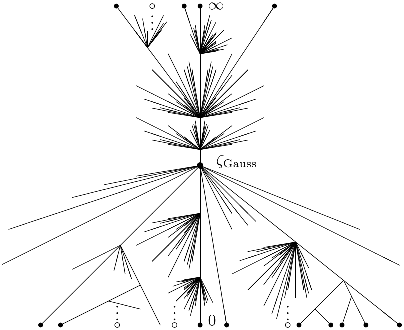

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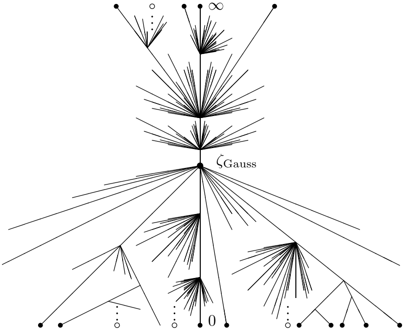

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Ⓓ $\overline{f} = (f_i)$ s.t. $K[X] = \langle f_i \rangle$:

$\mathrm{Trop}_{\overline{f}}(X) = \{v'(f_1), \ldots, v'(f_n) :$
$v' : K[X] \to \overline{\mathbb{R}}$ extends $v\}$

So Trop is projection of big space of v'. Can we reassemble it?

(semi)val gives (semi)norm:
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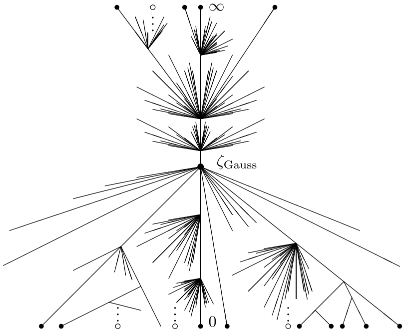
By [Pay08] we have:
"analytification is the limit of all tropicalizations"
(come to Stefano's talk for details)



Berkovich projective line, by Baker and Silverman:

Several types of points:

Type I: X embeds into them
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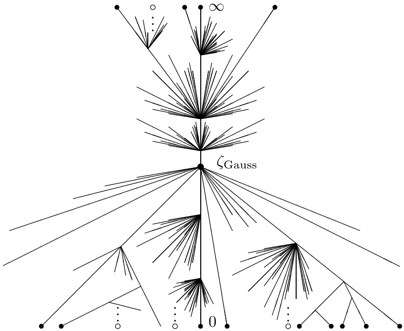

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Type IV: absent if K spherically complete *(no cauterizations)*

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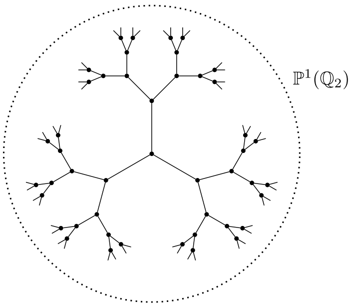

FIGURE 2. The affine Bruhat-Tits building $\overline{\mathcal{B}}_1(\mathbb{Q}_2)$

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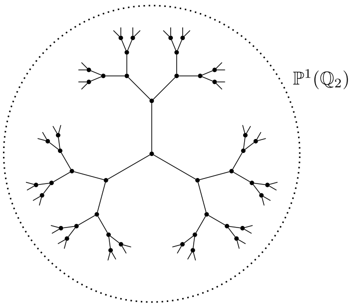

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What is $Trop(P^r, \iota)$?

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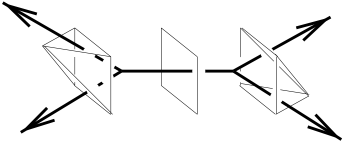

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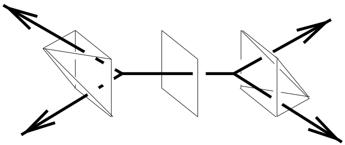

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References

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